



Modelling of COVID-19 Disease Spread in Yogyakarta City Using the Fourth-Order Runge Kutta Method and SIR model

Aditya Nur Pratama, Putu Harry Gunawan*

Faculty of Informatics, Departement of Informatics, Telkom University, Bandung
Jl. Telekomunikasi. 1, Terusan Buahbatu - Bojongsoang, Telkom University, Sukapura, Kec. Dayeuhkolot, Kabupaten Bandung, Jawa Barat, Indonesia

Email: ¹aditnp@student.telkomuniversity.ac.id, ²*phgunawan@telkomuniversity.ac.id

Correspondence Author Email: phgunawan@telkomuniversity.ac.id

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Abstract–The COVID-19 pandemic, caused by the SARS-CoV-2 virus, has created enormous worldwide health issues, particularly in Yogyakarta, Indonesia, a city with distinct socio-cultural dynamics and a crucial role in national education. Understanding how the virus spreads in this particular milieu is critical for successful public health responses. To simulate and investigate COVID-19 transmission dynamics in Yogyakarta, this work uses the Susceptible-Infected-Recovered (SIR) epidemiology model, enhanced by the Fourth Order Runge-Kutta (RK4) numerical approach. The RK4 technique improves the model's accuracy by providing precise numerical solutions to the differential equations governing disease transmission. The study identifies the optimal infection rate parameter ($\beta = 0.2037$) that minimizes the Root Mean Squared Error (RMSE) between the model's predictions and actual data. These findings offer critical insights into the local pandemic trajectory, which can directly support the government in tailoring public health strategies, assist researchers in refining epidemiological models, and guide the general public in understanding transmission risks. The methodologies and results from this study can also serve as a reference for similar epidemiological assessments in other regions.

Keywords: Covid-19; SIR-Model; Runge-Kutta; Yogyakarta; Disease

1. INTRODUCTION

The COVID-19 pandemic caused by the SARS-CoV-2 virus has become a serious global health issue. The virus was first discovered in Wuhan, China, in late 2019 and spread rapidly around the world. Indonesia, including the city of Yogyakarta, has also been severely affected by this epidemic. Each region requires a customized approach to understand and respond to the spread of this disease. Yogyakarta city with its rich culture, ethnic diversity and important role in national education provides a unique social identity. Factors such as population movement, population density, and level of access to health services may play a role in determining how the virus spreads and its impact on public health. Therefore, this study on the spread of COVID-19 in Yogyakarta City is not only useful but also important for disease control efforts[1]. Within this framework, this study adopts mathematical models, especially the SIR model, to understand and analyze the dynamics of the spread of COVID-19. The importance of numerical methods such as fourth-order Runge-Kutta in improving the accuracy of the model is crucial, allowing us to see daily changes in the number of cases, the severity of the disease, and the effectiveness of public health initiatives in Yogyakarta City[1]. The SIR model categorizes the population into three classes: susceptible, infected, and recovered[2]. However, to improve the fidelity and accuracy of the model, the use of numerical methods such as fourth-order Runge-Kutta is relevant. This approach provides consistent and accurate numerical solutions to differential equations, allowing for a more accurate representation of population dynamics during an epidemic.

In Yogyakarta City, the fourth-order Runge-Kutta method can facilitate a better understanding of the daily changes in the number of cases, severity of illness, and impact of public health initiatives[3]. It is intended to provide a scientific basis for broader policy and faster response to changes in local epidemiological profiles[4]. By combining the theory of the SIR model and the advantages of the fourth-order Runge-Kutta numerical method, this study aims to have a significant impact on the understanding and modeling of COVID-19 in the city of Yogyakarta. The findings of this research are expected to help the government, researchers, and other parties in developing more effective strategies in controlling this pandemic and reducing its negative impact on society and the local economy[1][5]. The significance of this research lies not only in understanding the dynamics of disease spread, but also in its impact on policy formulation. The research findings are expected to provide a foundation for smarter decision-making that is responsive to changes in the epidemiological profile at the local level. This research not only creates solutions to address COVID-19 in Yogyakarta, but also provides a basis for learning and adaptation in other cities across Indonesia and the world. Through this process, researchers hope that this research will not only benefit local communities in Yogyakarta, but also serve as a source of information for researchers in other cities across Indonesia and the world. With a deeper understanding of the factors that influence the spread of meningitis, it is hoped to provide responsive, efficient and effective health services for meningitis around the world.

The Susceptible-Infectious-Recovered (SIR) model has been a cornerstone of epidemiology since its introduction by Kermack and McKendrick in 1927. Classic works such as "Mathematical Epidemiology" provide an in-depth understanding of the basic concepts of this model[6]. Model, including delivery rate and recovery rate.



This study describes how the SIR model responds to various variables and how the model can be adapted to describe situations of epidemic spread in the community. Numerical methods, particularly the fourth Runge-Kutta method, are important in the handling of differential equations based on disease models. "Number Theory: The Art of Scientific Computation" introduces the basic principle of the fourth Runge-Kutta law and gives an explanation of its validity[7]. From this paper, this study can explain how to directly apply this numerical method to solve the differential equations of the SIR model[8]. The SIR model has been adopted to understand and predict the spread of COVID-19. "Epidemic model of the 2019 novel coronavirus disease (COVID-19) outbreak" provides an example of how to apply the SIR model in the context of the COVID-19 pandemic, taking into account specific parameters such as the reproduction rate (R_0) and other factors specific to the SARS-CoV-2 virus[8][9]. By combining the various parts of this paper, this research can provide a strong foundation for investigating the spread of COVID-19 in Yogyakarta City[1][10]. These resources not only include mathematical and statistical aspects, but also consider epidemiology, decision-making, and social issues in the local area. Therefore, this study is expected to provide a complete understanding of the disease situation and contribute greatly to the management of infectious diseases in the region[11].

Several studies have applied the SIR model and various numerical methods to analyze the spread of COVID-19 in different contexts. For instance, Chinazzi et al. (2020) developed a model to assess the impact of travel restrictions on the spread of COVID-19. Their study used a global metapopulation disease transmission model, which incorporated the SIR framework, to evaluate how restrictions on travel could slow the spread of the virus and delay its peak in various regions[12]. In another study, Kissler et al. (2020) explored the dynamics of SARS-CoV-2 transmission using an extended SIR model that included seasonal variation in transmission rates. Their findings suggested that recurrent wintertime outbreaks of COVID-19 were likely unless long-term immunity was achieved or effective vaccines were widely distributed[13]. Wang et al. (2020) analyzed the outbreak in Wuhan, China, using a combination of epidemiological modeling and actual case data. Their findings emphasized the crucial role of non-pharmaceutical interventions in controlling the spread of the virus and highlighted the challenges of predicting epidemic trends in a rapidly changing environment[14]. Tang et al. (2020) estimated the transmission risk of COVID-19 by applying the SIR model to data from the early outbreak phase. They identified key factors influencing the basic reproduction number (R_0), which is essential for determining the potential for outbreak control and guiding public health interventions[15]. Furthermore, Kuniya (2020) applied the SIR model to predict the peak of the COVID-19 epidemic in Japan. The study focused on the basic reproduction number and demonstrated the effectiveness of early and sustained interventions in mitigating the epidemic's impact[16]. While these studies provide a valuable foundation for understanding the utility of the SIR model and numerical methods in pandemic analysis, there are notable differences and gaps between this research and the existing literature. Previous studies have primarily focused on broad national or regional scales, whereas this research specifically targets Yogyakarta City, a medium-sized city with unique social and cultural dynamics[17]. Additionally, the use of the fourth-order Runge-Kutta method in this study offers a more accurate and stable solution to the differential equations within the SIR model, allowing for finer temporal resolution in predictions. This approach is particularly useful for assessing the daily impact of public health interventions in a localized setting, a gap not fully addressed in earlier research[18]. By combining the SIR model with the advantages of the fourth-order Runge-Kutta method, this study aims to contribute new insights into the spread of COVID-19 in Yogyakarta City. The findings are expected to assist local authorities and policymakers in crafting more effective strategies for disease control, thereby minimizing the pandemic's impact on both public health and the local economy[3].

2. RESEARCH METHODOLOGY

2.1 Research Stages

We created a research flow to help in the process of conducting this research. This flow covers the workflow for the research.

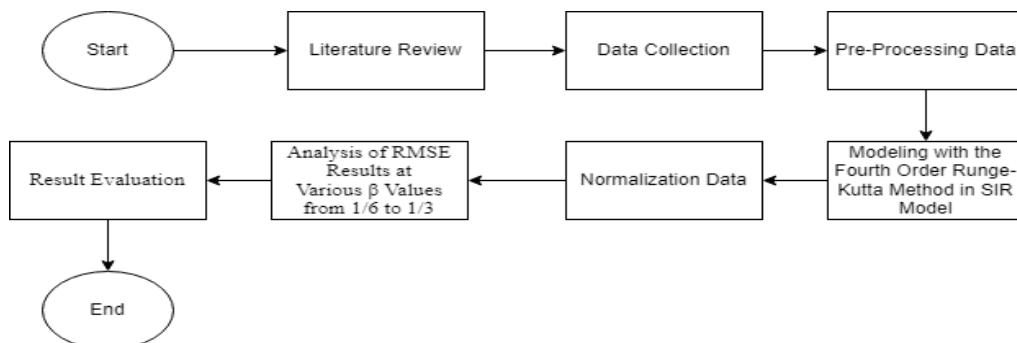


Figure 1. Research flow

The flowchart presented in Figure 1 the method starts with a thorough literature review. This process entails gathering and analyzing prior research, theories, models, and procedures pertinent to the topic. The goal of this evaluation is to lay a firm foundation and identify gaps or areas for more inquiry. delineates a structured approach for modeling and analyzing the spread of COVID-19 in Yogyakarta City, employing the SIR (Susceptible-Infected-Recovered) epidemiological model, with a particular focus on utilizing the Fourth Order Runge-Kutta (RK4) numerical method. The initial phase of the process involves the systematic gathering of relevant epidemiological data. Data sources is included health records website on the internet. Upon data acquisition, the next step involves pre-processing, a crucial phase aimed at ensuring data integrity and suitability for modeling. Following pre-processing, the SIR model is applied to the data, with the Fourth Order Runge-Kutta (RK4) method being employed to numerically solve the differential equations that govern the model. The results from the SIR model are then subjected to detailed analysis.

This analysis focuses on understanding the characteristics of the epidemic within Yogyakarta, such as the rate of infection spread, the peak number of infected individuals, and the duration of the outbreak. To ensure that the results of the analysis are meaningful and comparable, the data is normalized. Normalization adjusts the data to account for variations in population size, data reporting practices, and other factors that might otherwise distort the analysis. The next step is to examine the Root Mean Square Error (RMSE) results. This analysis is performed using values ranging from 1/6 to 1/3. The Root Mean Square Error (RMSE) is a typical method for measuring a model's error in predicting quantitative data. The study's goal is to discover the most accurate and dependable model configuration by analyzing RMSE at various values. The final stage of the process is the evaluation of the model's results. This involves comparing the model's predictions against actual observed data to assess its accuracy.

2.2 Research Method

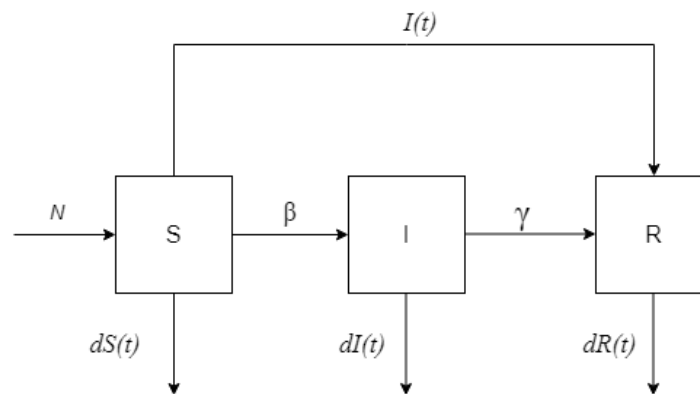


Figure 2. Schematic of the COVID-19 SIR Model in Yogyakarta

The scheme of the figure 2 above presents the interpopulation transmission of the COVID-19 case in Yogyakarta. The scheme also illustrates the SIR model, which is used to simulate a susceptible (S), infected (I), and recovered (R) population in each variable that represents the group's population count. Individuals are vulnerable in the beginning and can become infected as a result of the COVID-19. This causes a drop in the number of vulnerable populations while increasing the number of infected populations. The same process happens when an infected population recovers and moves to the recovered population, causing the infected population to drop and the recovered population to increase. Some of the evaluation metrics that will be conducted to measure the performance of the system such as:

1. Equation for Susceptible population, $S(t)$:

$$\frac{dS(t)}{dt} = -\beta S(t)I(t) \tag{1}$$

2. Equation for the infected population, $I(t)$:

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t) \tag{2}$$

3. Equation for the recovered population, $R(t)$:

$$\frac{dR(t)}{dt} = \gamma I(t) \tag{3}$$

The $S(t)$, $I(t)$, $R(t)$: The number of susceptible, infected, and recovered individuals at time t , respectively. N is the equal of Total population, β is the equal of Contact rate that results in transmission, γ is the equal of Recovery rate, The N is constant and can be expressed as: $N = S(t) + I(t) + R(t)$. The fourth-order Runge-Kutta method (RK4) can be applied to any differential equation to obtain its numerical solution. For one variable (e.g. $S(t)$), the RK4 steps can be written as follows

$$K1 = \Delta t \cdot f(t, S(t), I(t), R(t)) \tag{4}$$

$$K2 = \Delta t \cdot f\left(t + \frac{\Delta t}{2}, S(t) + \frac{K1}{2}, I(t) + \frac{K1}{2}, R(t) + \frac{K1}{2}\right) \tag{5}$$

$$K3 = \Delta t \cdot f\left(t + \frac{\Delta t}{2}, S(t) + \frac{K2}{2}, I(t) + \frac{K2}{2}, R(t) + \frac{K2}{2}\right) \tag{6}$$

$$K4 = \Delta t \cdot f(t + \Delta t, S(t) + K3, I(t) + K3, R(t) + K3) \tag{7}$$

Where $F(t), S(t), I(t), R(t)$ are the functions that represent the differential equation and Δt is the time interval. After calculating $K1, K2, K3,$ and $K4$ the variable values can be updated as follows:

$$S(t + \Delta t) = S(t) + \frac{1}{6}(K1 + 2K2 + 2K3 + K4) \tag{8}$$

$$I(t + \Delta t) = I(t) + \frac{1}{6}(K1 + 2K2 + 2K3 + K4) \tag{9}$$

$$R(t + \Delta t) = R(t) + \frac{1}{6}(K1 + 2K2 + 2K3 + K4) \tag{10}$$

The same detailed and methodical approach can be extended to the calculation of $I(t)$, representing the infected population over time, and $R(t)$, representing the recovered or removed population. By applying this process to these variables, the model achieves a holistic and comprehensive evaluation of the epidemic's progression. This method stands out as a particularly effective and precise numerical approach for determining how the population evolves at each discrete time step. The core of this approach is the Runge-Kutta 4th order method, commonly abbreviated as RK4, which is widely recognized in numerical analysis for its balance between computational efficiency and accuracy. The RK4 method involves a sophisticated process of calculating four intermediate "steps" during each iteration: $K1, K2, K3,$ and $K4$. Each of these steps represents a successive approximation that progressively refines the estimation of the solution at each time step. $K1$ is calculated based on the initial slope, while $K2, K3,$ and $K4$ are determined by using estimates of the slope at points within the time step, incorporating the effect of earlier estimates.

These steps are not simply averaged; they are combined with carefully determined weights that account for the nonlinear nature of the equations being solved. This weighted combination leads to an update of the numerical solution that is significantly more accurate than what could be achieved with simpler methods, such as the Euler method or even the midpoint method. The advantage of using the RK4 method in this context is that it provides a high degree of accuracy without requiring prohibitively small steps, which would increase computational costs. This is crucial when modeling complex systems like the spread of infectious diseases, where precision in predicting the number of susceptible, infected, and recovered individuals over time can make a significant difference in understanding and managing the outbreak. In the context of studying the spread of COVID-19 in Yogyakarta City, the RK4 method allows researchers to numerically analyze the disease's dynamics by incorporating key epidemiological factors, such as the contact rate (which influences how quickly the disease spreads through the population) and the recovery rate (which determines how quickly individuals move from the infected to the recovered category).

3. RESULT AND DISCUSSION

3.1 Diagram Plot

The figure 3 below illustrates the cumulative progression of COVID-19 cases in the Special Region of Yogyakarta up to January 15, 2024. The graph provides a visual representation of the total number of individuals affected by the virus over this period. The vertical axis of the figure, which ranges from zero to 300,000, indicates the number of persons who have been impacted by COVID-19, including those who have been infected, recovered, or succumbed to the disease. The horizontal axis represents the timeline, beginning on March 14, 2020, and extending through to January 15, 2024.

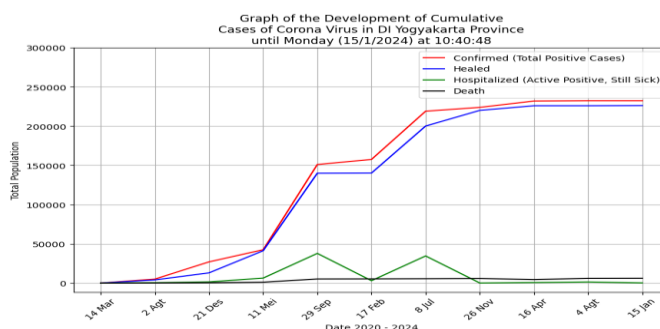


Figure 3. Actual Data of COVID-19 Diagram in Yogyakarta

The total number of confirmed COVID-19 cases is shown by the red line on the graph, which experiences a notable spike in late December and then stabilizes close to the top, which could be attributed to various factors, such as holiday travel, large gatherings, or the emergence of a new variant. After this spike, the red line begins to level off, indicating that the rate of new infections has slowed down as it approaches the upper limit of the graph. The red line is closely followed by the blue line, which shows the number of recoveries, suggesting that the majority of confirmed cases have made a full recovery this high recovery rate is a positive indicator of the effectiveness of the healthcare response in the region, as well as the resilience of the population. The near-parallel movement of the blue line with the red line, particularly after the initial months of the pandemic, reflects a system where recovery rates have kept pace with the rise in cases, preventing the healthcare system from being overwhelmed. The green line reflects the number of active instances in which persons are still receiving treatment. It peaks at particular times before dropping abruptly, indicating that the number of active cases varies over time. The gray line, which represents the number of deaths, is either invisible or very faint, indicating a low death toll in this dataset. Overall, the graph shows a considerable increase of COVID-19 cases, a high recovery rate, oscillations in active cases, and a low mortality toll in the Yogyakarta region over the selected time period.

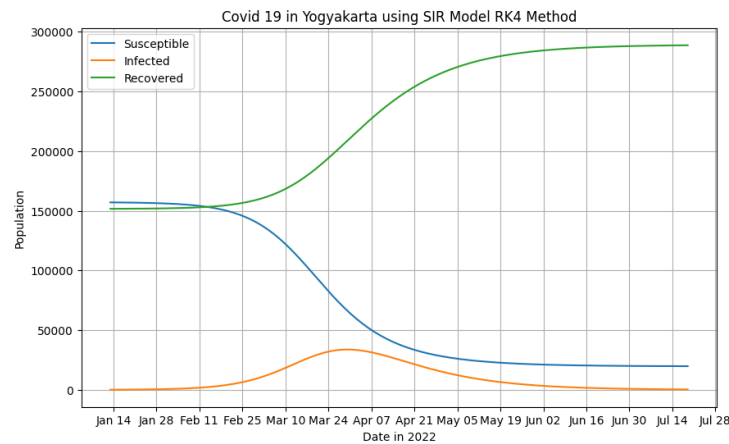


Figure 4. Plot of COVID-19 Diagram in Yogyakarta using SIR Model RK4 method

The spread of COVID-19 within Yogyakarta is intricately modeled and depicted in Figure 4. This simulation emerges from a comprehensive analysis using the Fourth-Order Runge-Kutta method (RK4), combined with the SIR (Susceptible, Infected, Recovered) epidemiological model. The parameters chosen for this analysis are $\beta = 1/3$, representing the transmission rate, and $\gamma = 1/14$, denoting the recovery rate. These parameters are critical in determining the dynamics of the disease spread within the modeled population. The total population considered in this simulation is 300,000 individuals. Out of this entire population, 157,059 individuals are categorized as susceptible to the disease at the onset of the simulation. Additionally, 107 individuals are identified as currently infected with COVID-19, while a significant portion, 151,684 individuals, have already recovered from the infection as the simulation begins [19].

The Susceptible group, represented by the blue curve in the graph, consists of the large number of individuals within the population who have not yet contracted COVID-19 but are at risk of infection due to their lack of immunity. The progression of this group is one of the key elements in understanding the spread of the virus. Initially, almost the entire population is classified as susceptible, highlighting the potential for widespread transmission of the disease. The Infected group, illustrated by the orange curve, represents the number of individuals who are actively infected with COVID-19 at any given time during the simulation. At the beginning of the simulation, the number of infected individuals is relatively low, reflecting the early stages of the outbreak. However, as time progresses, the orange curve shows a rapid and sharp increase in the number of infections, signifying the acceleration of the disease's spread within the community. The Recovered group is depicted by the green curve, which represents the number of individuals who have successfully recovered from COVID-19 infection. The green curve starts with a fairly substantial number of recovered individuals, as a significant portion of the population had already recovered from the virus prior to the beginning of the simulation. This initial condition reflects the ongoing nature of the pandemic, where a certain segment of the population has already experienced the infection and developed immunity. Through the application of the SIR model, combined with the computational precision of the RK4 method, this simulation effectively captures the complex dynamics of how an epidemic unfolds within a closed population. The model provides a vivid illustration of the entire lifecycle of the epidemic—from the initial spread of the virus through the susceptible population, to the rapid increase in infections, and finally to the eventual decline as the population transitions to a state of recovery. This simulation underscores the importance of understanding the interplay between susceptible, infected, and recovered individuals in managing and predicting the course of an epidemic, particularly in the context of public health interventions and policy-making. Through the application of the SIR model, combined with the computational precision of the

RK4 method, this simulation effectively captures the complex dynamics of how an epidemic unfolds within a closed population. The model provides a vivid illustration of the entire lifecycle of the epidemic—from the initial spread of the virus through the susceptible population, to the rapid increase in infections, and finally to the eventual decline as the population transitions to a state of recovery. This simulation underscores the importance of understanding the interplay between susceptible, infected, and recovered individuals in managing and predicting the course of an epidemic, particularly in the context of public health interventions and policy-making.

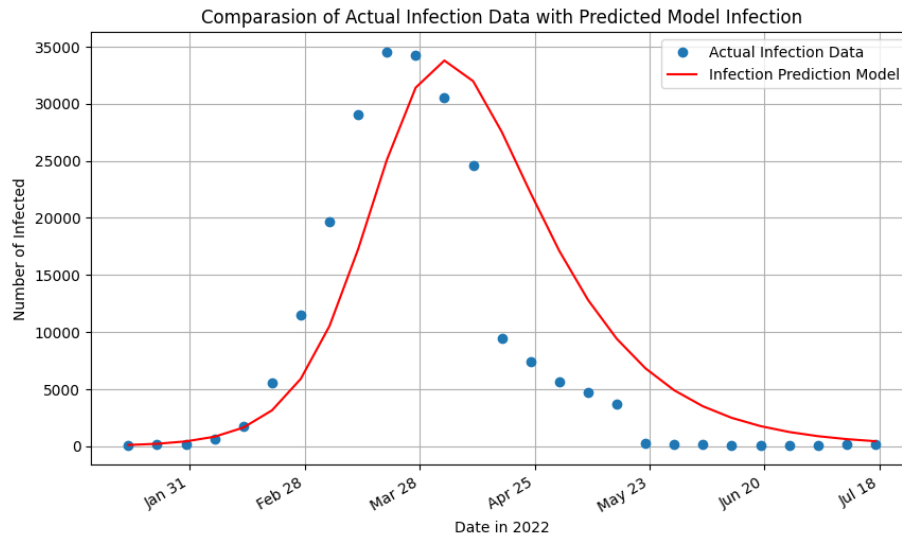


Figure 5. Plot Diagram of Comparison of Actual Infection Data with Prediction Model Infection

The figure 5 above indicates the outcomes of the assessment among the actual COVID-19 data in Yogyakarta and the prediction outcomes the use of the SIR version with parameters $\beta = 1/3$ and $\gamma = 1/14$ the use of the Runge-Kutta 4th order method (RK4). The red line indicates the expected variety of infections the use of the SIR while blue dots represent actual Covid-19 infection data observed in the field, version with $\beta = 1/3$ and $\gamma = 1/14$ This curve indicates the expected variety of inflamed people primarily based totally at the SIR version simulation. The curve additionally indicates a rising withinside the variety of infections, achieving a peak, after which reducing over time[20].

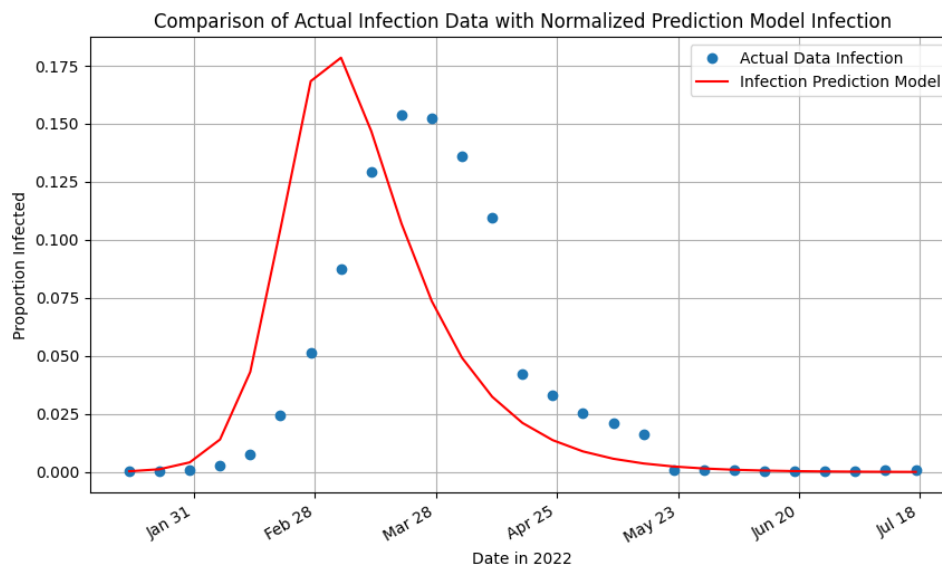


Figure 6. Plot Diagram of Comparison of Actual Infection Data with Normalized Prediction Model Infection

The figure 6 above presented illustrates the comparison between the actual data of COVID-19 infection in Yogyakarta and the predicted results of the normalized SIR (Susceptible-Infected-Recovered) model. The SIR model uses a transmission rate parameter β of $1/3$ and a recovery rate γ of $1/14$, with a numerical solution obtained through the 4th order Runge-Kutta method (RK4). This graph shows that the SIR model, despite its limitations in capturing real-time variations from actual data, is still quite effective in predicting the general trend of Covid-19 spread in Yogyakarta. Differences that occur at the peak of infection may be caused by external factors that are not modeled, such as public health interventions, changes in individual behavior, or virus mutations.



3.2 Analysis of RMSE Results at Various Values

In this study, a simulation of the SIR (Susceptible-Infected-Recovered) model was carried out to the model the spread of COVID-19 in Yogyakarta using the 4th order Runge-Kutta method (RK4). The parameter we used in this simulation is the value of β (infection rate) within a range of 1/6 to 1/3. Simulation results are compared with actual weekly infection data to calculate the Root Mean Squared Error (RMSE) which describes the difference between model predictions and actual data. The results of the RMSE calculation for various β values are shown in the table below.

Table 1. The RMSE result for each coefficient infection rates.

| No | The coefficient infection rate (β) | RMSE |
|----|--|--------|
| 1 | 1/6 | 0.0327 |
| 2 | 1/5.4 | 0.0156 |
| 3 | 1/4.9 | 0.0138 |
| 4 | 1/4.5 | 0.0266 |
| 5 | 1/4.2 | 0.0390 |
| 6 | 1/3.9 | 0.0494 |
| 7 | 1/3.6 | 0.0578 |
| 8 | 1/3.4 | 0.0646 |
| 9 | 1/3.2 | 0.0702 |
| 10 | 1/3 | 0.748 |

From the table 1 above, it becomes evident that the parameter value $\beta = 0.2037$ yields the lowest Root Mean Square Error (RMSE) value of 0.0138. This low RMSE value indicates that, at this particular value of β , the SIR model provides the most accurate prediction of the infection data when compared to the actual observed data. The accuracy of the model at this β value suggests that it closely mirrors the real-world spread of COVID-19 in Yogyakarta, making it a highly reliable parameter for simulating the progression of the disease. In contrast, the highest RMSE value is observed when $\beta = 0.3333$, with an RMSE of 0.0748. This significantly higher RMSE value implies that the model's predictions deviate more substantially from the actual infection data at this β value. The larger discrepancy between the predicted and actual data suggests that the assumed infection rate in the model is less representative of the true dynamics of COVID-19 spread within the population. Essentially, as β increases to 0.3333, the model overestimates the rate at which individuals move from the susceptible to the infected category, leading to predictions that are less aligned with the observed data. The variation in RMSE values across different β parameters highlights the sensitivity of the SIR model to the infection rate parameter. A noticeable drop in the RMSE value at the specific β value of 0.2037 suggests that the infection rate utilized in the model at this point is closely aligned with the actual rate of COVID-19 transmission in Yogyakarta.

This alignment between the model and the observed data is critical, as it ensures that the predictions made by the model are not only accurate but also reflective of the real-world situation. Therefore, $\beta = 0.2037$ can be considered the most appropriate parameter value for modeling the spread of the infection, as it provides the best fit between the model's predictions and the actual data. The implications of this finding are significant for both the study and broader public health applications. By identifying $\beta = 0.2037$ as the optimal infection rate parameter, this study offers valuable insights into the dynamics of disease transmission that can be applied to real-world scenarios. The results of this research can be utilized to fine-tune the SIR model, ensuring that it more accurately simulates the spread of infectious diseases like COVID-19. This, in turn, can inform pandemic prevention and management strategies, providing decision-makers with reliable data to base their actions on. For instance, knowing the most appropriate infection rate parameter can help in predicting the future course of the pandemic, assessing the potential impact of public health interventions, and optimizing resource allocation to mitigate the spread of the disease. Furthermore, the methodology and findings of this study can serve as a framework for future research in modeling infectious diseases. The approach of systematically varying the β parameter and analyzing its impact on the RMSE values can be replicated in other contexts, allowing for the identification of optimal parameters in different populations or for different diseases. By refining the model to closely match the actual dynamics of disease spread, researchers and policymakers can improve the accuracy of their predictions and enhance the effectiveness of their responses to public health crises.

4. CONCLUSION

In conclusion, this study has successfully integrated the Susceptible-Infected-Recovered (SIR) model with the Fourth Order Runge-Kutta (RK4) numerical method to extensively examine the dynamics of COVID-19 transmission in Yogyakarta, Indonesia. The combination of these approaches created a strong foundation for simulating the virus's propagation, capturing crucial features of the pandemic's evolution such as infection rates, peak infection periods, and subsequent recovery phases. The RK4 method was especially useful, as it improved accuracy in solving the differential equations inherent in the SIR model, allowing for a more thorough and reliable



simulation of the disease's progression. This study's primary contribution is identifying the infection rate parameter (β) that best fits the observed data for Yogyakarta. $\beta = 0.2037$ yields the lowest Root Mean Squared Error (RMSE) value, indicating the highest predicting accuracy. This discovery is critical since it not only improves the model's dependability in Yogyakarta but also serves as a useful reference point for similar epidemiological investigations in other places. The ability to effectively forecast and anticipate the spread of COVID-19 is critical for effective public health planning and intervention, especially in areas with distinct demographic and cultural characteristics like Yogyakarta. Furthermore, the study emphasizes the need of using advanced numerical approaches, such as RK4, in epidemiological modeling. While traditional models provide a basic insight, the use of sophisticated numerical approaches enables a more detailed analysis that can account for the intricacies of real-world events. This technique not only enhances the model's prediction precision, but it also allows researchers and policymakers to explore multiple scenarios and confidently estimate the potential impact of various public health actions. In conclusion, this study not only increases our understanding of COVID-19 transmission in Yogyakarta, but also sheds light on the broader application of mathematical models in epidemiology. The combination of the SIR model and the RK4 technique provides a strong tool for forecasting and managing the spread of infectious illnesses, having consequences for both local and worldwide public health initiatives. The findings highlight the necessity of ongoing study and innovation in this sector, especially as we work to build more effective and responsive tactics for countering future pandemics.

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